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# FEATURES OF IN-PHASE SYNCHRONIZATION IN THE MODEL OF COUPLED NEURONS WITH DIFFERENT TYPES OF COUPLING

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**Abstract.** In the present paper a model of coupled neuron with excitory and inhibitory coupling is considered. It is shown that the in-phase solution appears as a result of a sequence of period-doubling bifurcations of the degenerate period-1 cycle. Multistability between in-phase and out-of-phase solutions is shown in the system.

**Keywords:** coupled neuron model, synchronization, inhibitory and exciting coupling.

### Introduction

The basis of the functioning and interaction of neurons is the central rhythm generators. Typical for such systems is that during the interaction its demonstrate an antiphase synchronization [1-2]. In recent papers [3-6] it was shown that the interaction of different types of coupling (excitory and inhibitory) between systems may cause synchronization. It has also been shown that multistability between inphase and antiphase oscillations is possible in such a system.

The purpose of this paper is to analyze the synchronization between two coupled models of neurons, to study the multistability between in-phase and antiphase regimes.

## 1. Object of investigation, typical dynamical regime.

As an object of research, we will use the phenomenological Hindmarsh-Rose model, which describes the main features of neuron dynamics, in which the multistability effect between in-phase and antiphase oscillations was shown in [3-6].

The system of two identical coupled neurons with excitatory and inhibitory

coupling [1] can be written in the following form:

$$\dot{x}_{i} = ax_{i}^{2} - x_{i}^{3} - y_{i} - z_{i} + g_{exc}k_{exc}(V_{exc} - x_{i})\Gamma(x_{j}) + g_{inh}k_{inh}(V_{inh} - x_{i})\Gamma(x_{j}) 
\dot{y}_{i} = (a + \alpha)x_{i}^{2} - y_{i}, 
\dot{z}_{i} = \mu(bx_{i} + c - z_{i})$$
(1)

i, j = 1, 2, where x is the membrane potential, the variables y and z characterize the transfer of ions through the membrane through the "fast" and "slow" ion channels, respectively. Fast synaptic coupling is modeled by sigmoid function:

$$\Gamma(x_j) = 1/[1 + \exp\{-\lambda(x_j - \Theta_s)\}]. \tag{2}$$

Here  $\Theta_s$  is the synaptic threshold,  $\Theta_s = -0.25$  [7].  $V_{exc}$ ,  $V_{inh}$  are the thresholds of the exciting and inhibitory couplings. Accordingly, for  $V_{exc} = 2 > x_i(t)$  and  $V_{inh} = -2 < x_i(t)$  for any  $x_i(t)$  there is an exciting and inhibitory coupling. Constant parameters are chosen and fixed as follows: a = 2.8,  $\alpha = 1.6$ ,  $\lambda = 10$ , c = 5, b = 9,  $\mu = 0.001$ , which corresponds to chaotic burst-spiking dynamics in a single neuron model [3-7].

In the present paper we consider the case of a symmetric mutual coupling of two pulse-coupled generators with attractive and repulsive coupling, which is provided by the parameters  $k_{\rm exc} = 1$  and  $k_{\rm inh} = 1$  in (1). Synaptic parameters  $g_{\rm exc}$  and  $g_{\rm inh}$ , determine the strength of the exciting and inhibitory coupling, respectively.

## 2. Features of synchronization and multystability.

It was shown in [3-6] that with the variation of the coupling parameters, different types of dynamics are possible: in-phase and antiphase synchronous oscillations, chaotic oscillations, periodic oscillations. To analyze the dynamics of the system and the bifurcation mechanisms of the creation of synchronization and complex oscillatory regimes, we turn to a one-parameter bifurcation analysis. Figure 1 shows the bifurcation trees for the model (1) with a fixed exciting coupling (gexc = 0.5) and the variation of the braking coupling (ginh). For the purpose of analyzing and detecting multistability in the system, the trees were calculated with continuation choosing of the initial conditions, and also with a different scanning of the control parameter. Figure 1a shows in black and red the projection of the tree on the dynamic variable z1 in the Poincaré section by the hypersurface x1 = 0. Thus, Figure 1a allows identification of coexisting regimes and multistability areas, the corresponding intervals are indicated by the symbol M in the Figure. Thus, two intervals are clearly visible, where multistability is observed. With a small coupling, these are coexisting antiphase and in-phase oscillations. The in-phase mode is generated on the basis of a cycle of period-1, as a result of a sequence of period-doubling bifurcations, a in-phase chaotic regime appears from the cycle.

Figure 1b shows two projections of the same tree on the dynamic variable z1 and the variable z2, which allow approximately to determine the in-phase and antiphase solutions. The corresponding areas are marked in the figure with the letters S and A, respectively.

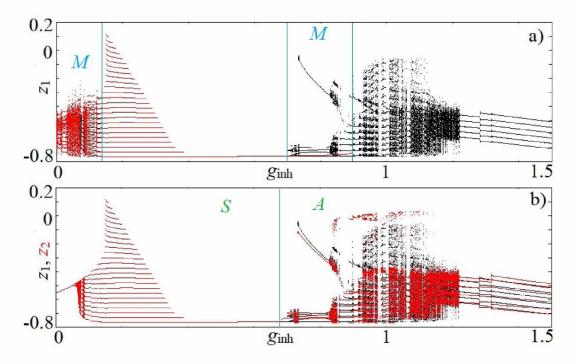


Figure 1 – Bifurcation trees a) projections of the variable x1, constructed with continuation chossing of the initial conditions when scanning the parameter interval from left to right (red color) and right to left (black color), M are areas where multistability is observed; b) projection x1 (black color) and x2 (red color), S mark in-phase oscillations, A marks antiphase oscillations.

#### Conclusion

Thus, within the framework of this paper it is shown that in the model of coupled neurons with an exciting and inhibitory coupling, in-phase and antiphase synchronization may occur. It is shown that the in-phase solution appears as a result of a sequence of period-doubling bifurcations of the degenerate cycle period-1. Multistability between in-phase and out-of-phase solutions is possible in the system.

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